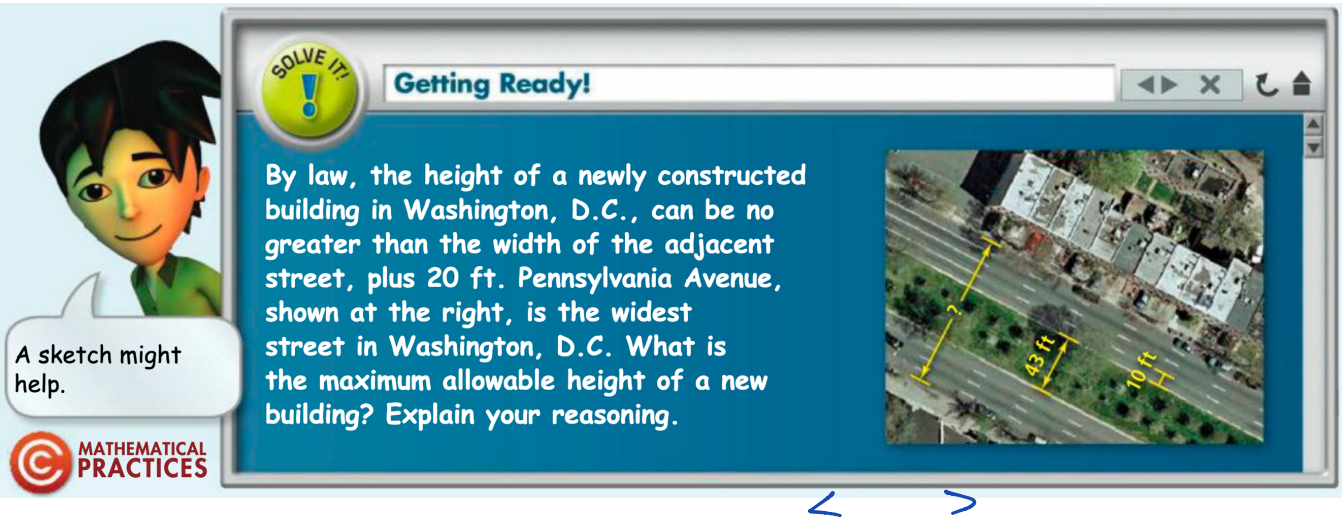
3-1 Inequalities and Their Graphs



$<$ $>$ $\leq$ $\geq$

An inequality is a mathematical sentence that uses an inequality symbol to compare the values of two expressions. You can use a number line to visually represent the values that satisfy an inequality.

PROBLEM 1: WRITING INEQUALITIES

What inequality represents the verbal expression?

*a) all real numbers **x** less than or equal to -7*

$x \leq -7$

*b) 6 less than a number **k** is greater than 13*

$k - 6 > 13$

*c) **v** is greater than or equal to 5*

$v \geq 5$

*d) the quotient of **k** and 9 is greater than 2*

$\frac{k}{9} > 2$

*e) **b** is less than 4*

$b < 4$

Careful
b less than 4
4 - b

*f) 3 less than **g** is less than or equal to 17*

$g - 3 \leq 17$

*g) all real numbers **p** greater than or equal to 1.5*

$p \geq 1.5$

*h) the sum of **t** and 7 is less than -3*

$t + 7 < -3$

A ***solution of an inequality*** *is any number that makes the inequality true.* The solutions of the inequality $x < 5$ are all real numbers *x* that are less than 5. You can evaluate an expression to determine whether a value is a solution of an inequality.

$3 \geq 3$
yes

PROBLEM 2: IDENTIFYING SOLUTIONS BY EVALUATING

Is the number a solution of $2x + 1 > 3$*?*

a) -3

$2(-3)+1 \stackrel{?}{>} 3$
$-6+1 > 3$
$-5 \not> 3$
NO

b) -1

$2(-1)+1 \stackrel{?}{>} 3$
$-2+1 > 3$
$-1 > 3$
No

c) 1

$2(1)+1 \stackrel{?}{>} 3$
$2+1 > 3$
$3 > 3$
no

d) *Consider the numbers -1, 0, 1, and 3. Which are solutions of* $13 - 7y \leq 6$*?*

$y=-1$; $13-7(-1) \leq 6$
$13+7 \leq 6$
$20 \leq 6$
NO

$y=0$; $13-7(0) \leq 6$
$13-0 \leq 6$
$13 \leq 6$
No

$y=1$; $13-7(1) \leq 6$
$13-7 \leq 6$
$6 \leq 6$
yes

$y=3$; $13-7(3) \leq 6$
$13-21 \leq 6$
$-8 \leq 6$
yes

Determine whether each number is a solution of the given inequality.

e) $4m - 6 \leq 10$

i) 2

$4(2)-6 \leq 10$
$8-6 \leq 10$
$2 \leq 10$
yes

ii) 0

$4(0)-6 \leq 10$
$0-6 \leq 10$
$-6 \leq 10$
yes

iii) 5

$4(5)-6 \leq 10$
$20-6 \leq 10$
$14 \leq 10$
No

f) $\frac{6-n}{n} \geq 11$

i) 0.5

$\frac{6-0.5}{0.5} \geq 11$
$\frac{5.5}{0.5} \geq 11$
$11 \geq 11$
yes

ii) 2

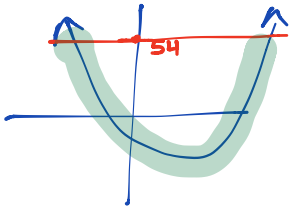
$\frac{6-2}{2} \geq 11$
$\frac{4}{2} \geq 11$
$2 \geq 11$
No

iii) 4

$\frac{6-4}{4} \geq 11$
$\frac{2}{4} \geq 11$
$\frac{1}{2} \geq 11$
NO

g) $m(m-3) < 54$

$m^2 - 3m < 54$



i) -10

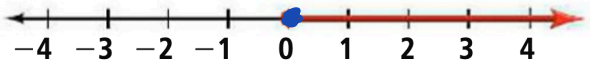
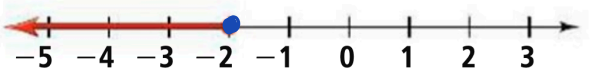
$-10[(-10)-3] < 54$
$-10(-13) < 54$
$130 < 54$
NO

ii) 0

$0[(0)-3] < 54$
$0 < 54$
yes

iii) 9

$9[(9)-3] < 54$
$9(6) < 54$
$54 < 54$
No

Inequality	Graph	
$n < 1$		The open dot shows that 1 is *not* a solution. Shade to the left of 1.
$a \geq 0$		The closed dot shows that 0 is a solution. Shade to the right of 0.
$f > -3$		The open dot shows that -3 is *not* a solution. Shade to the right of -3.
$-2 \geq x$ $x \leq -2$		The closed dot shows that -2 is a solution. Shade to the left of -2.

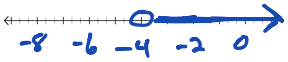
You can also write $-2 \geq x$ as $x \leq -2$.

PROBLEM 3: GRAPHING AN INEQUALITY

Graph each inequality.

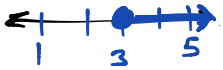
$2\frac{1}{4}$
$4\overline{)9}$
-8
1

a) $x > -4$



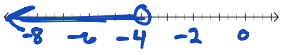
b) $c < 0$

c) $3 \leq n$ or $n \geq 3$



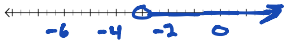
d) $c \geq -\frac{9}{4}$ $c \geq -2\frac{1}{4}$ $c \geq -2.25$

e) $t < -4$

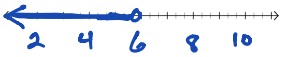


f) $v \geq 2$

g) $-3 < f$ or $f > -3$



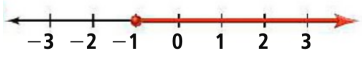
h) $5.75 > d$ or $d < 5.75$



PROBLEM 4: WRITING AN EQUATION FROM A GRAPH

What inequality represents the graph?

a)

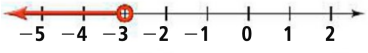


$a \geq -1$

b)

$b < 4$

c)

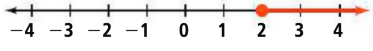


$x < -3$

d)

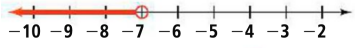
$d \geq 0$

e)



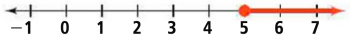
$e \geq 2$

f)



$f < -7$

g)



$y \geq 5$

h)

$h < 2$

PROBLEM 5: WRITING REAL-WORLD INEQUALITIES

What inequality describes the situation? Be sure to define a variable.

a)

Let c = cost of a trail ride

$c \geq 19.99$

b)

Let s = legal speed

$0 \leq s \leq 8$

Define a variable and write an inequality to model each situation.

c) The restaurant can seat at most 172 people.

Let p = people $p \leq 172$

d) A person must be at least 35 years old to be elected President of the United States.

Let a = age $a \geq 35$

e) A light bulb can be no more than 75 watts to be safely used in this light fixture.

Let w = watts $w \leq 75$

f) At least 475 students attended the orchestra concert Thursday night.

Let s = students $s \geq 475$

g) A law clerk has earned more than $20,000 since being hired.

Let e = earnings $e > 20000$